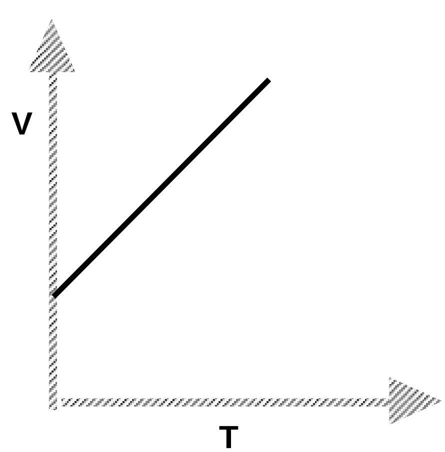
Unit I Reading – Graphical Methods

One of the most effective tools for the visual evaluation of data is a graph. The investigator is usually interested in a quantitative graph that shows the relationship between two variables in the form of a curve.

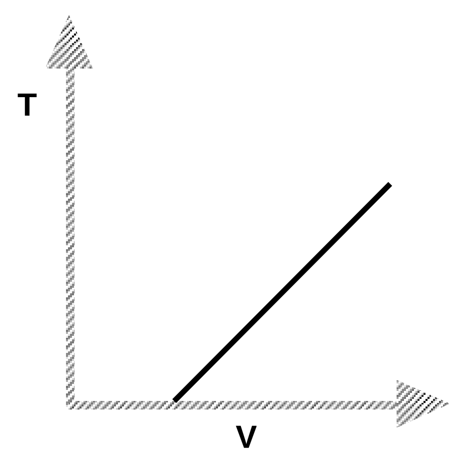
For the relationship y = f(x), **x** is the *independent variable* and **y** is the *dependent variable*. The rectangular coordinate system is convenient for graphing data, with the values of the dependent variable y being plotted along the *vertical axis* and the values of the independent variable x plotted along the *horizontal axis*.

Positive values of the dependent variable are traditionally plotted above the origin and positive values of the independent variables to the right of the origin. This convention is not always adhered to in physics, and thus the positive direction along the axes will be *indicated by the direction the arrow heads point.*

The choice of dependent and independent variables is determined by the experimental approach or the character of the data. Generally, the **independent variable** is the one over which the *experimenter has complete control*; the **dependent variable** is the one that *responds to changes* in the independent variable. An example of this choice might be as follows. In an experiment where a given amount of gas expands when heated at a constant pressure, the relationship between these variables, V and T, may be graphically represented as follows:



Correct



Incorrect

By established convention it is proper to plot V = f (T) rather than T = f (V), since the experimenter can directly control the temperature of the gas, but the volume can only be changed by changing the temperature.

A. ***Curve Fitting***

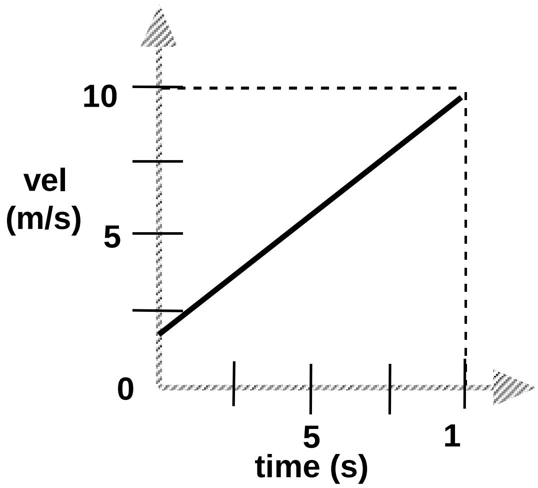
When checking a law or determining a functional relationship, there is good reason to believe that a uniform curve or straight line will result. The process of matching an equation to a curve is called **curve fitting**. The desired empirical formula, assuming good data, can usually be determined by inspection. There are other mathematical methods of curve fitting, however they are very complex and will not be considered here. Curve fitting by inspection requires an assumption that the curve represents a linear or simple power function.

If data plotted on rectangular coordinates yields a straight line, the function y = f(x) is said to be *linear* and the line on the graph could be represented algebraically by the slope-intercept form:

$$y = mx + b,$$

where **m** is the slope and **b** is y-intercept.

Consider the following graph of velocity vs. time:



The curve is a straight line, indicating that v = f(t) is a linear relationship. Therefore,

$$\mathbf{v} = m\mathbf{t} + b,$$

where slope $= m = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

From the graph,

$$m = \frac{8.0 \text{ m/s}}{10.0 \text{ s}} = 0.80 \text{ m/s}^2 .$$

The curve intercepts the v-axis at v = 2.0 m/s. This indicates that the velocity was 2.0 m/s when the first measurement was taken; that is, when t = 0. Thus, $b = v_0 = 2.0$ m/s.

The general equation, **v** = m**t** + b, can then be rewritten as

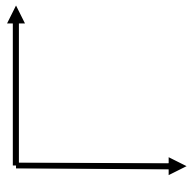
$$\mathbf{v} = (0.80 \text{ m/s}^2)\mathbf{t} + 2.0 \text{ m/s}.$$

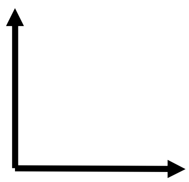
Worksheet 1: GRAPHING PRACTICE

For each data set below, determine the mathematical expression. To do this, first graph the original data. Assume the 1st column in each set of values to be the **independent** variable and the 2nd column the **dependent** variable. Write an appropriate mathematical expression for the relationship between the variables. <u>Be sure to include units!</u>

Data set 1

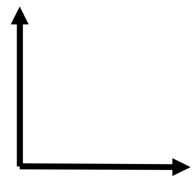
V (m^3)	**P (pa)**
.1	40
.5	8
1	4
2	2
4	1
5	.8
8	.5
10	.4

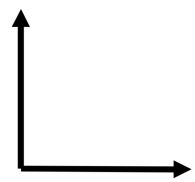




Data set 2

t (s)	**x (m)**
.1	.03
.2	.12
.5	.75
1	3
2	12
3	27
4	48
5	75





Mathematical expression #1

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Mathematical expression #2

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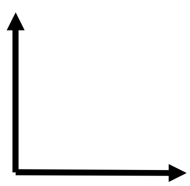
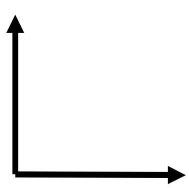
Data set 3

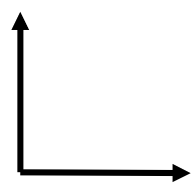
A (months)	**W (lbs)**
1	7.3
2	9.4
3	10.5
4	12.0
5	13.0
6	14.3
7	15.2
8	16.7

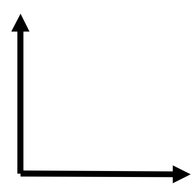




Data set 4

t (s)	**v (m/s)**
.3	10
1.2	20
2.7	30
4.8	40
7.5	50
10.8	60
14.7	70
19.2	80





Mathematical expression #3

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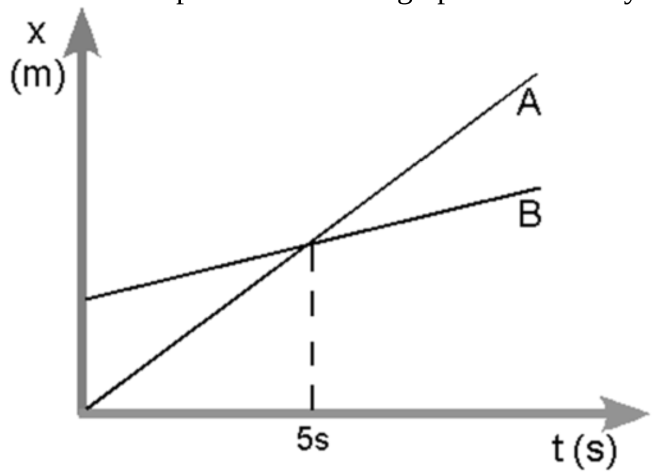
Mathematical expression #4

____

<u>Additional required reading for this unit is chapter 2 in Conceptual Physics. It is the expectation that you**</u>**
<u>read this before you come to class.**</u>**

Worksheet 2

1. Consider the position vs. time graph below for cyclists A and B.



a. Do the cyclists start at the same point? How do you know? If not, which is ahead?

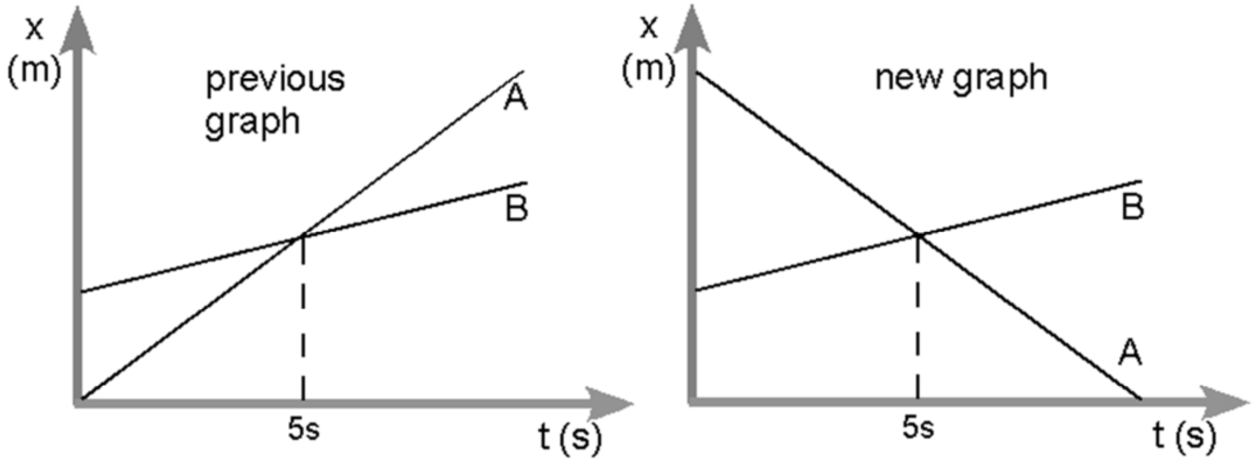
b. At t= 7s, which cyclist is ahead? How do you know?

c. Which cyclist is travelling faster at t = 3s? How do you know?

d. Are their velocities equal at any time? How do you know?

e. What is happening at the intersection of lines A and B?

2. Consider the new position vs. time graph below for cyclists A and B.



a. How does the motion of the cyclist A in the new graph compare to that of A in the previous graph from page one?

b. How does the motion of cyclist B in the new graph compare to that of B in the previous graph?

c. Which cyclist has the greater speed? How do you know?

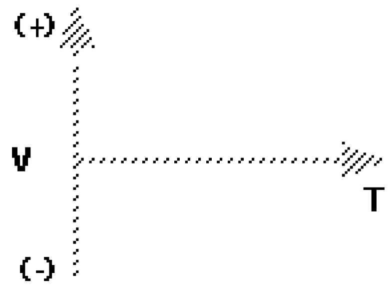
d. Describe what is happening at the intersection of lines A and B.

e. Which cyclist traveled a greater distance during the first 5 seconds? How do you know?

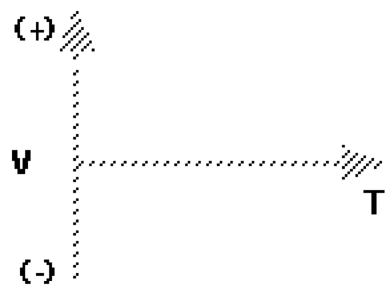
Worksheet 3

Sketch velocity vs time graphs corresponding to the following descriptions of the motion of an object.

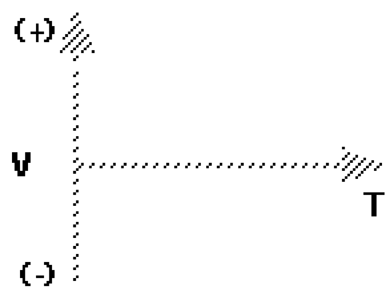
1. The object is moving away from the origin at a constant (steady) speed.



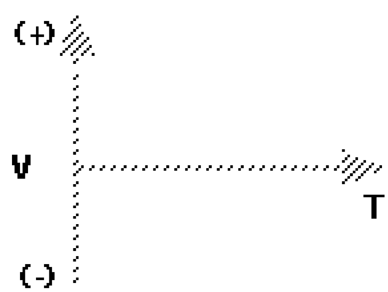
2. The object is standing still.



3. The object moves toward the origin at a steady speed for 10s, then stands still for 10s.

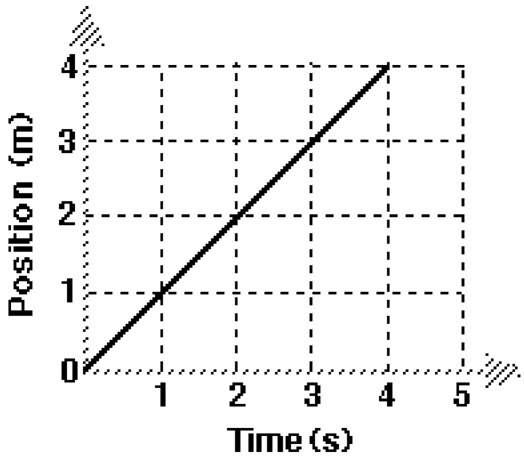


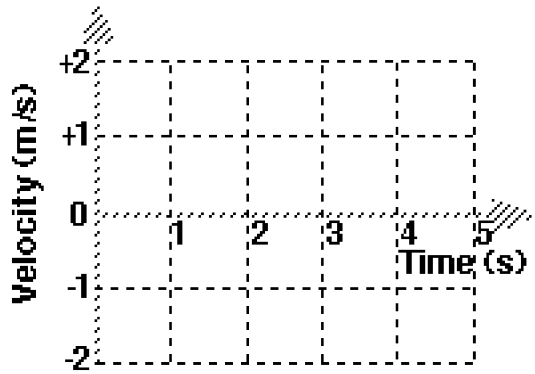
4. The object moves away from the origin at a steady speed for 10s, reverses direction and moves back toward the origin at the same speed.



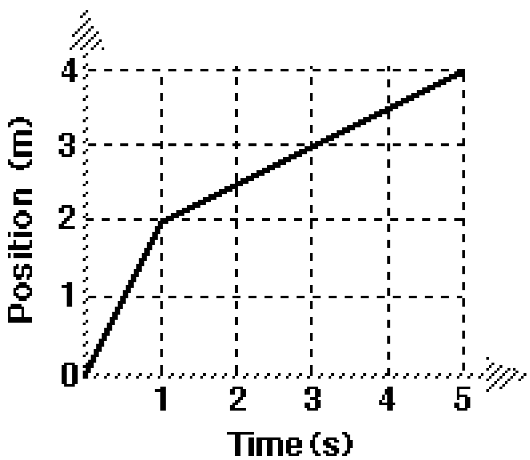
Draw the velocity vs time graphs for an object whose motion produced the position vs time graphs shown below at left.

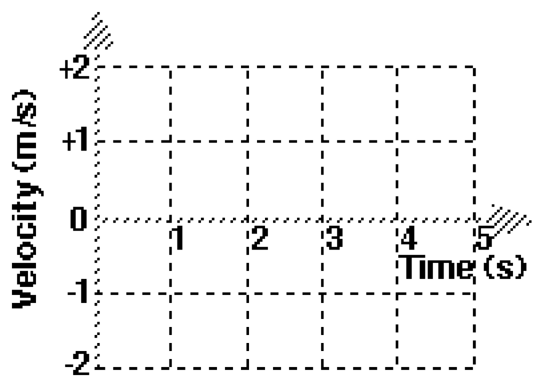
5.


Position (m)
0
1
2
3
4
1
2
3
4
5
Time (s)

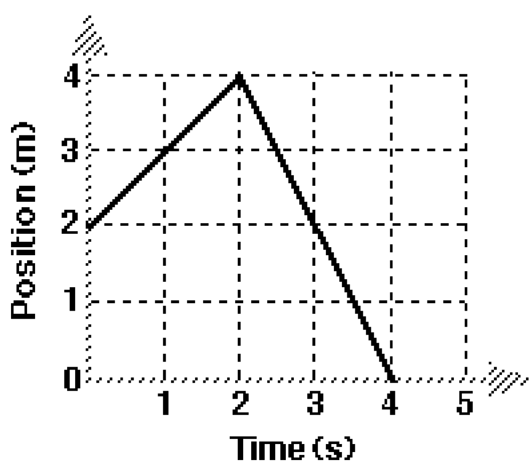

Velocity (m/s)
+2
+1
0
-1
-2
1
2
3
4
5
Time (s)

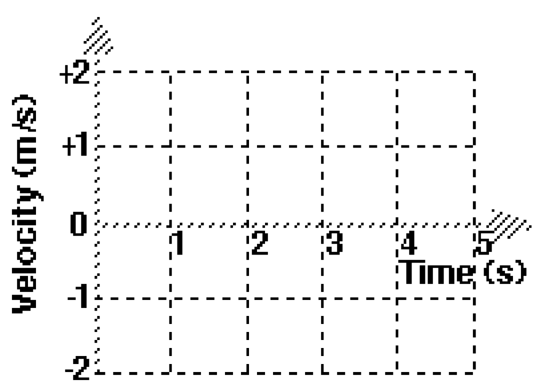
6.


Position (m)
0
1
2
3
4
1
2
3
4
5
Time (s)


Velocity (m/s)
+2
+1
0
-1
-2
1
2
3
4
5
Time (s)

7.

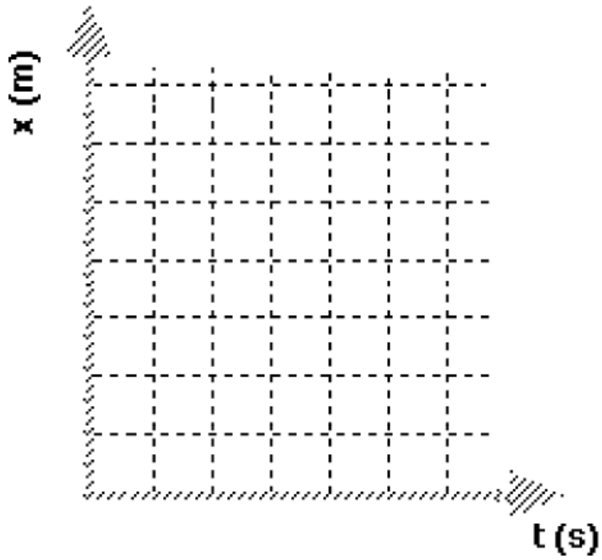

Position (m)
0
1
2
3
4
1
2
3
4
5
Time (s)


Velocity (m/s)
+2
+1
0
-1
-2
1
2
3
4
5
Time (s)

Worksheet 4

1. Robin, roller skating down a marked sidewalk, was observed to be at the following positions at the times listed below:

t (s)	x (m)
0.0	10.0
1.0	12.0
2.0	14.0
5.0	20.0
8.0	26.0
10.0	30.0



a. Plot a position vs. time graph for the skater.

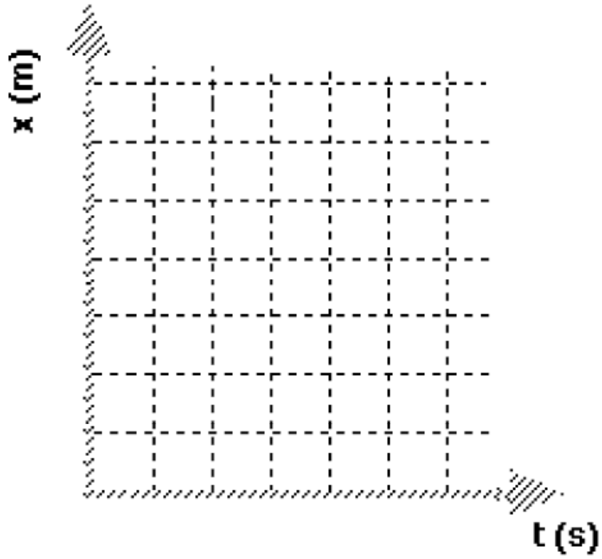
b. How far from the starting point was he at t = 6s? How do you know?

c. Write a mathematical model to describe the curve in (a).

d. Was his speed constant over the entire interval? How do you know?

2. The following data were obtained for a second trial:

t (s)	x (m)
0.0	4.0
2.0	10.0
4.0	16.0
6.0	22.0
8.0	28.0
10.0	34.0



a. Plot the position vs. time graph for the skater.

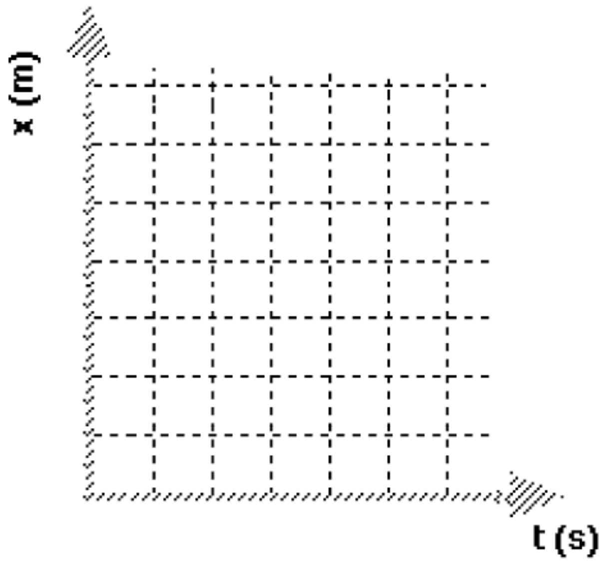
b. How far from the starting point was he at t = 5s? How do you know?

c. Was his speed constant? If so, what was it?

d. In the first trial the skater was further along at 2s than he was in the second trial. Does this mean that he was going faster? Explain your answer.

3. Suppose now that our skater was observed in a third trial. The following data were obtained:

t (s)	x (m)
0.0	0.0
2.0	6.0
4.0	12.0
6.0	12.0
8.0	8.0
10.0	4.0
12.0	0.0



a. Plot the position vs. time graph for the skater.

b. What do you think is happening during the time interval: t = 4s to t = 6s? How do you know?

c. What do you think is happening during the time interval: t = 6s to t = 12s? How do you know?

d. Determine the skater's average **speed** from t = 0s to t = 12s.

e. Determine the skater's average **velocity** from t = 0s to t = 12s.

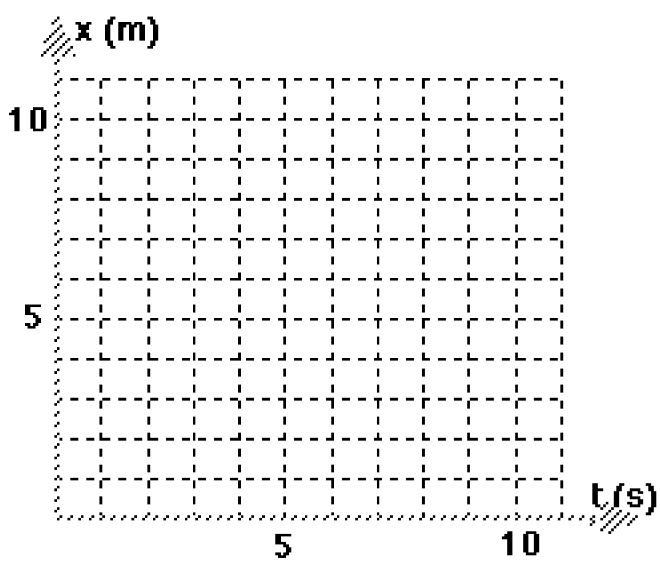
Worksheet 5

2. From the position vs time data below, answer the following questions.

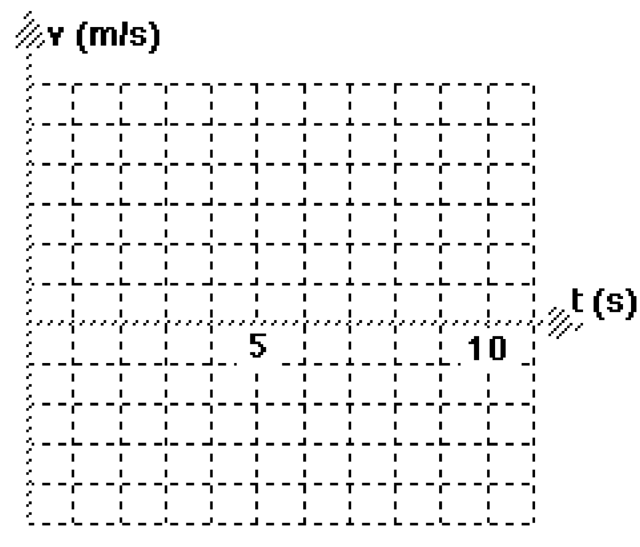
t (s)	x (m)
0	0
1	2
2	4
3	4
4	7
5	10
6	10
7	10
8	5
9	0

a. Construct a graph of position vs time.

b. Construct a graph of velocity vs time.



(A)

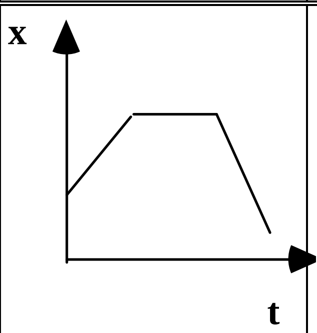
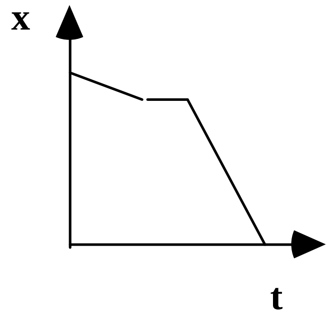
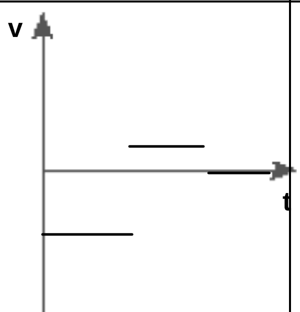
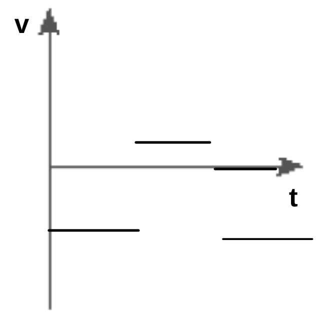


(B)

d. Determine the displacement from t = 3.0s to 5.0s using graph B.

e. Determine the displacement from t = 7.0 s to 9.0 s using graph B.

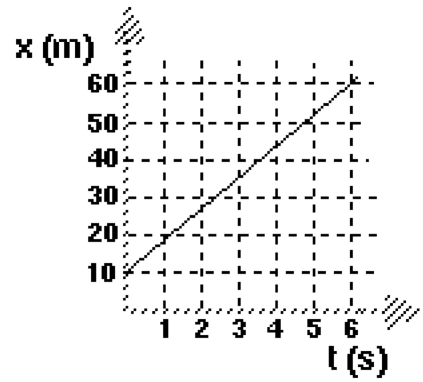
Worksheet 6

	1	2	3	4
x vs. t graph				
v vs. t graph				
Written Description				

	5	6
x vs. t graph		
v vs. t graph		
Written Description	Object moves with constant positive velocity for 4 seconds. Then, it stops for 2 seconds and returns to the initial position in 2 seconds.	Object A starts 10m to the right of the origin and moves to the left at 2 m/s. Object B starts at the origin and moves to the right at 3m/s.

Review

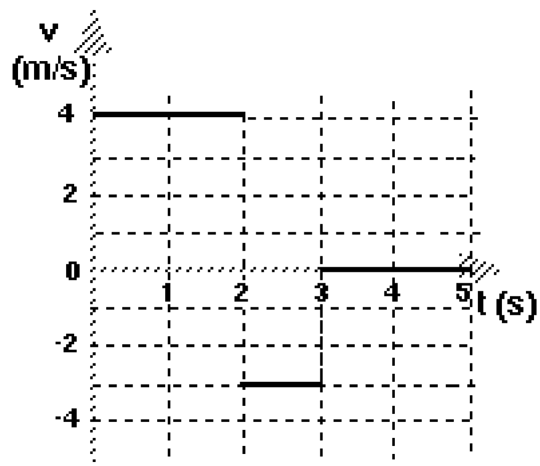
1. Consider the position vs time graph at right.



   a. Determine the average velocity of the object.

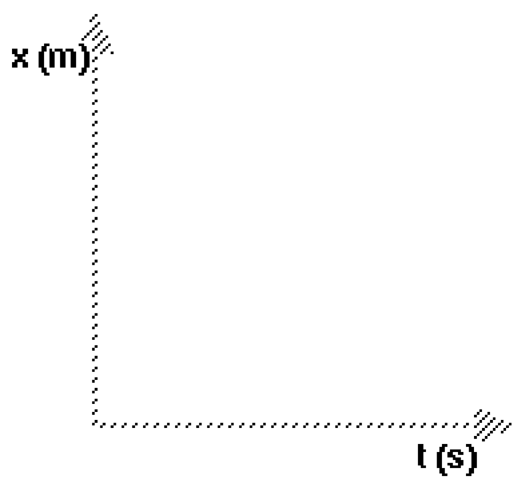
   b. Write a mathematical equation to describe the motion of the object.

2. Shown at right is a velocity vs time graph for an object.



   a. Describe the motion of the object.

   b. Draw the corresponding position vs time graph. Number the x - axis.



   c. How far did the object travel in the interval t =1s to t =2s?

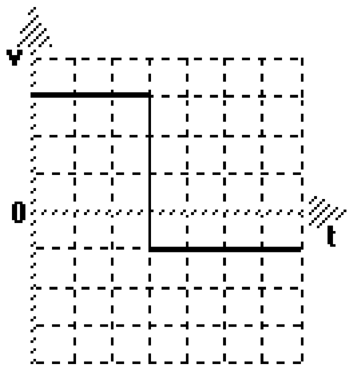
   d. What is the total displacement? Explain how you got the answer.

3. Johnny drives to Wisconsin (1920 miles) in 32 hours. He returns home by the same route in the same amount of time.
   a. Determine his average speed.

   b. Determine his average velocity.

   c. Compare these two values and explain any differences.

4. Consider the v vs t graph below.



   a. Describe the behavior of the object depicted in the graph.

5. A race car travels at a speed of 95 m/s. How far does it travel in 12.5 s? Use the appropriate mathematical expression and show how units cancel.

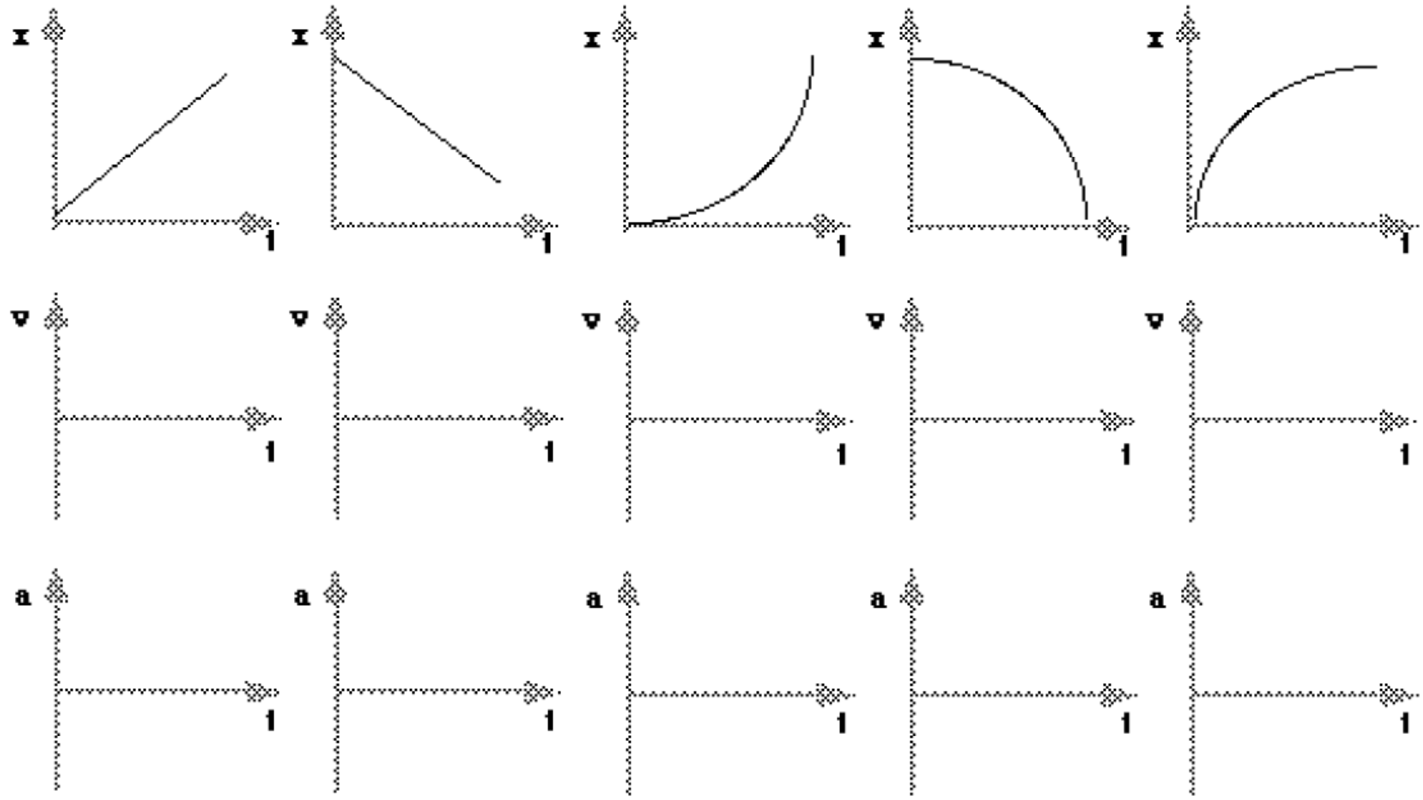
6. A ball rolls up a hill with a velocity of 2.5 m/s. It is being accelerated with a constant $-0.4\ m/s^2$. What is its velocity after 3.0 s?

7. A race car slows from 50 m/s to 22.3 m/s in 2.8 s. What is the cars acceleration?

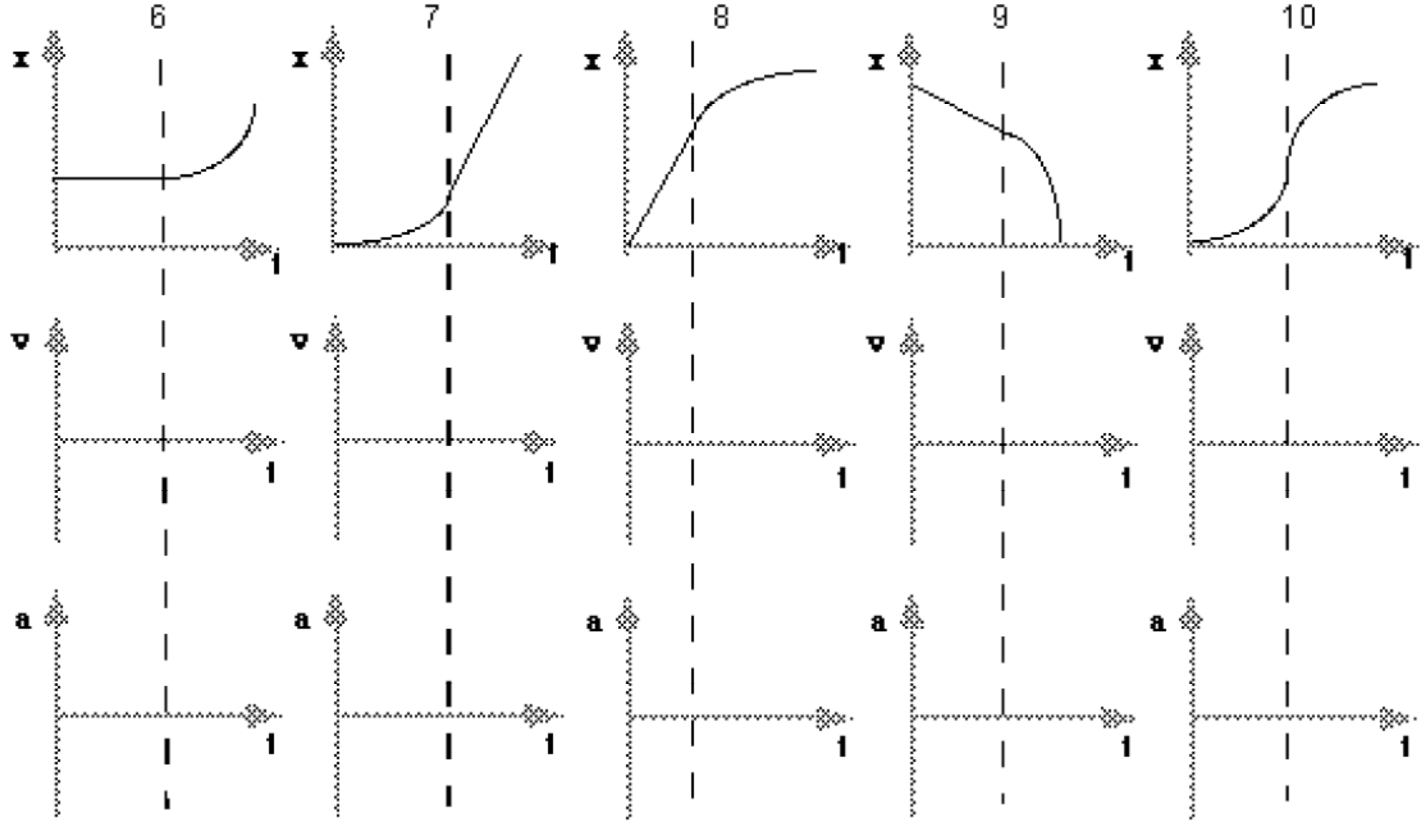
Worksheet 7: Stacks of kinematics curves

Given the following position vs time graphs, sketch the corresponding velocity vs time and acceleration vs time graphs.

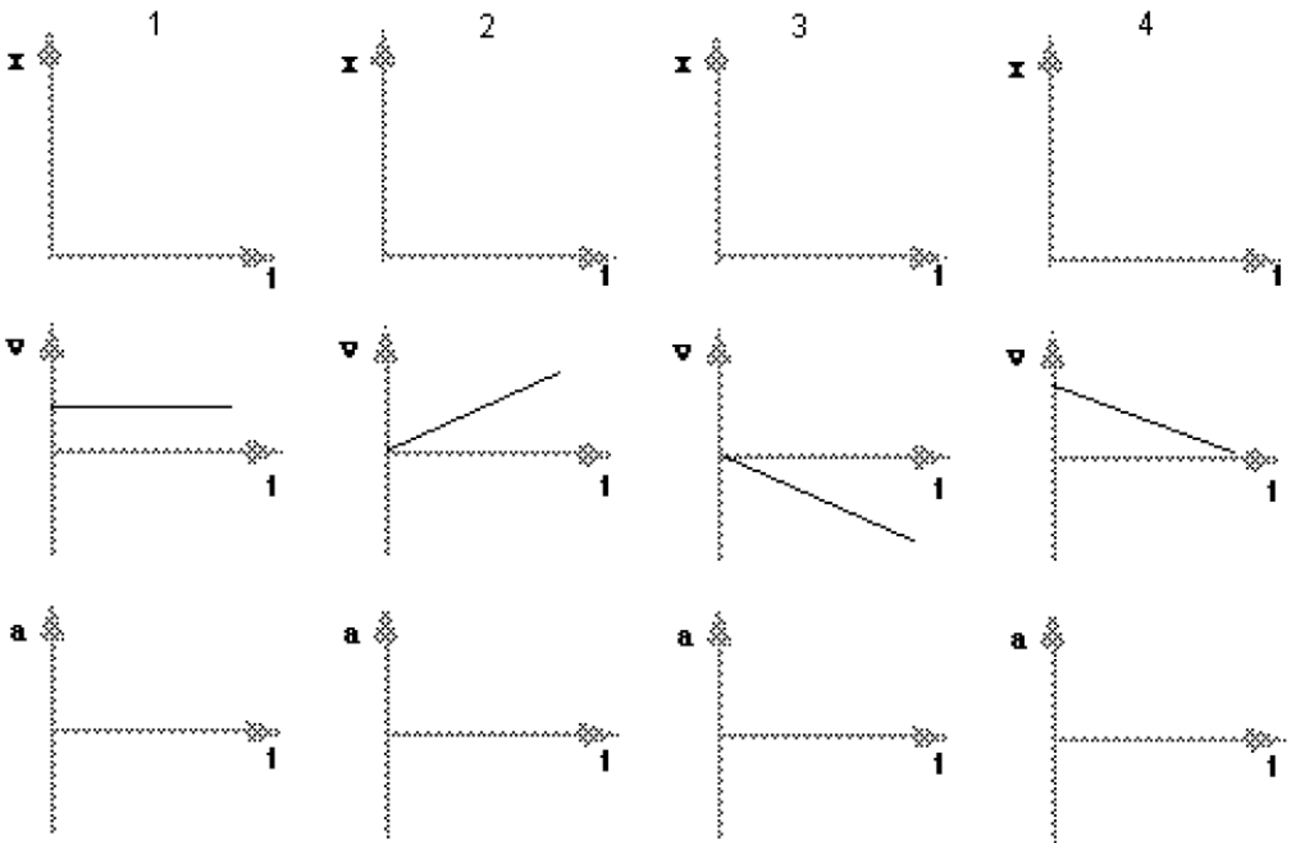
1 2 3 4 5

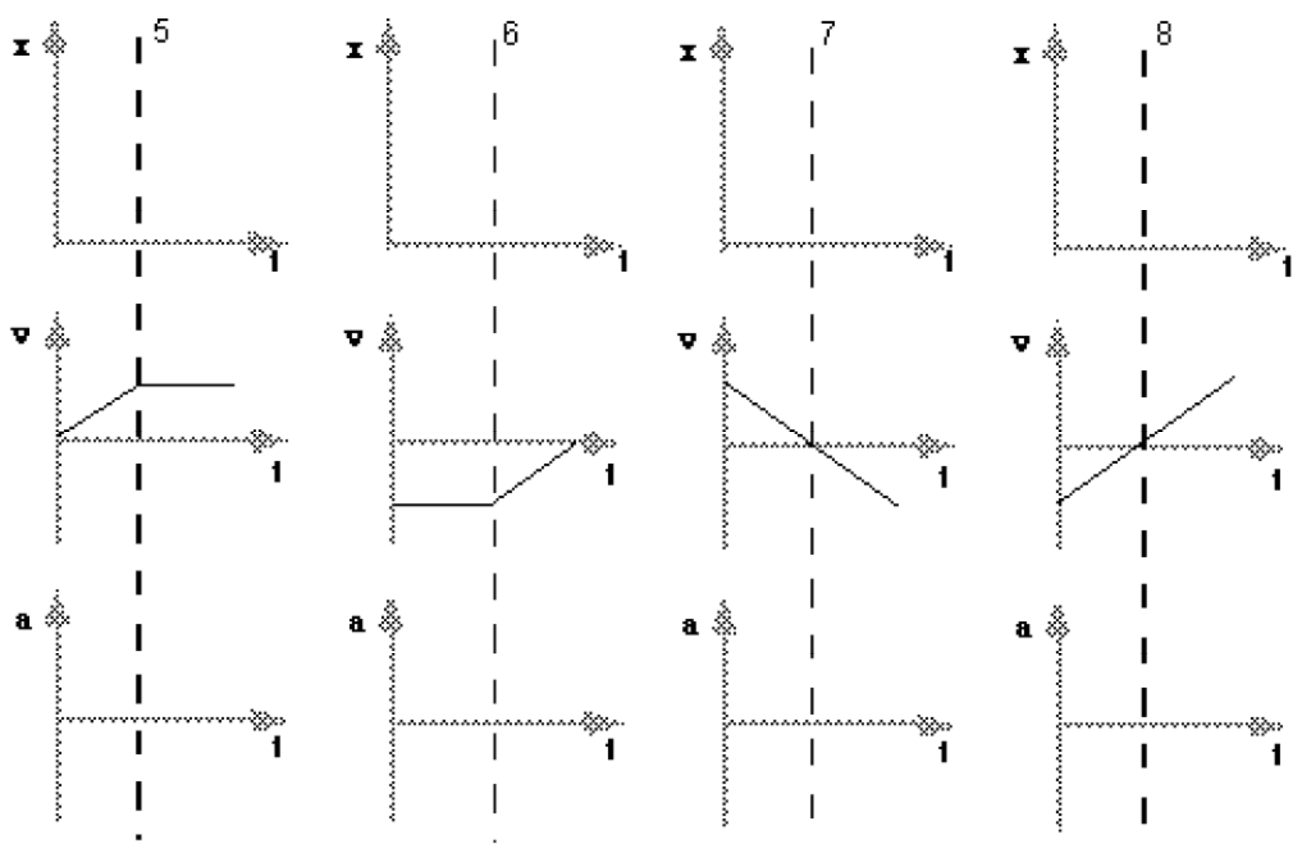


6 7 8 9 10



For the following velocity vs. time graphs, draw the corresponding position vs. time and a-t graphs

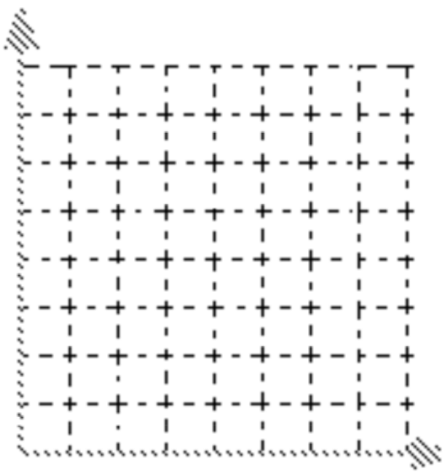
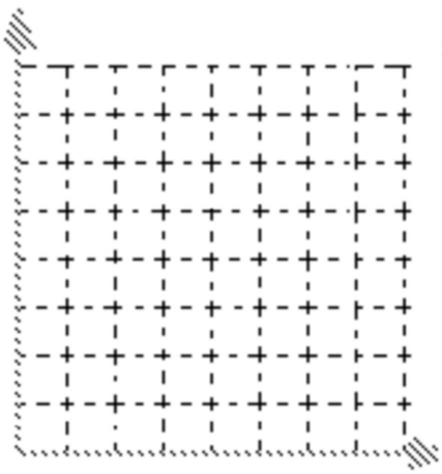
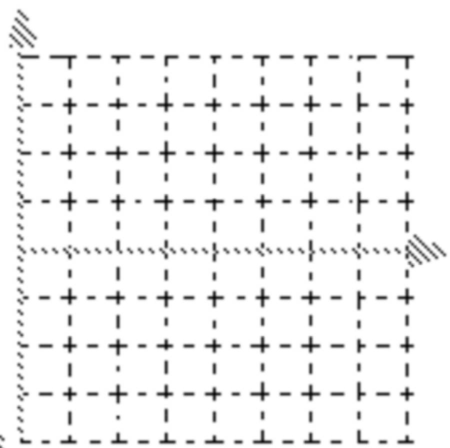




Worksheet 8

While cruising along a dark stretch of highway with the cruise control set at 25 m/s (≈55 mph), you see, at the fringes of your headlights, that a bridge has been washed out. You apply the brakes and come to a stop in 4.0s. *Assume the clock starts the instant you hit the brakes.*

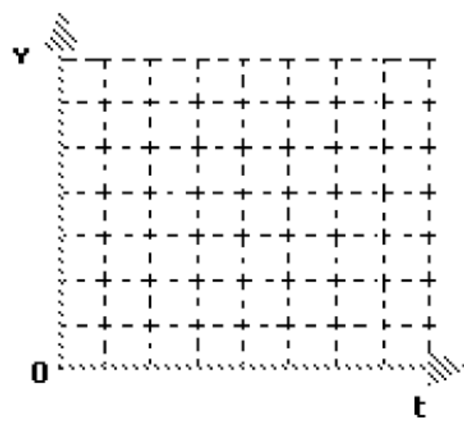
1. Construct **qualitative** graphical representations of the situation described above to illustrate:
   a. **x** vs. **t**
   b. **v** vs. **t**
   c. **a** vs. **t**

3. Construct a **quantitatively accurate v** vs **t** graph to describe the situation.

4. On the **v** vs **t** graph at right, graphically represent the car's displacement during braking.

5. Utilizing the **graphical representation**, determine how far the car traveled during braking. (Please explain your problem solving method.)



6. In order to draw the **a** vs **t** graph, you need to determine the car's acceleration. Please do this, then sketch a **quantitatively accurate a** vs **t** graph